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Measuring How Relatively "Good" a Hot-spot Map Is: A Summary of Current Metrics

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Abstract

Hot-spot maps are used by a majority of police departments throughout the United States. These maps are used to determine policing decisions such as community resource allocation and police presence. There are various methods to generate these maps; however, there is no consensus on when each specific mapping technique is best to use. We argue this is due to a lack of understanding of how "good" a hot-spot map is relative to another. Many data scientists use statistical metrics to evaluate hot-spot maps, while many police departments and hot-spot software use indices developed in the criminology literature. This paper bridges the gap between these fields by advancing the mathematical understanding of recent criminology hot-spot indices. We create a standard mathematical notation for hot-spot indices and explore the mathematical intuition and knapsack problem inherent in evaluating the most recent index, the Prediction Efficiency Index* (PEI*). We conclude with some directions where the evaluation of hot-spot maps might go in the future.

Keywords
Hot-spot mapping; knapsack application; criminal justice; spatiotemporal metrics

1. Introduction
Most police departments use hot-spot maps to identify crime patterns [1]. These crime hot-spot maps are used to allocate resources within various types of policing efforts, such as problem-orientated policing (e.g., enhancing location characteristics), community-oriented policing (e.g., improving community dynamics), and traditional policing activities (e.g., increasing or concentrating police presence) [2]. There are various methods to create these crime hot-spot maps, from naïve models to advanced machine learning algorithms. However, the literature is conflicting as to which model is the best; some argue Random Forest (RF), Kernel-Density Estimation (KDE), Multilayer Perceptron (MLP), and Risk Terrain Modeling (RTM) are among the most promising methods [3, 4]. A lack of consistent terminology, evaluation criteria, and reporting of initial parameters are some reasons behind the lack of consensus [3]. Since crime hot-spot maps have become central to various police and community resources decisions, it is critical to clearly communicate the metrics used to evaluate and compare maps. We begin by summarizing the two separate research directions in developing metrics to evaluate hot-spot maps: statistical metrics used in machine learning and computer science literature and crime indices used by police and criminologists. We then recontextualize current criminal justice crime indices into a consistent mathematical notation. Lastly, we identify the optimization model behind a recent crime index to aid the development of new metrics.

2. Narrative Review of Hot-spot Evaluation Metrics and Indices
When comparing hot-spot maps and measuring how well these maps perform, some have used general statistical measures for evaluations, while others have used specialized indices developed from the criminology literature. Hot-spot mapping studies in computer science and machine learning have used two types of statistical measures. The first is binary classification which measures how well an algorithm predicted a crime would occur in areas classified as 'hot' or 'not-hot' in a previous time period. Common binary classification metrics include Accuracy, Precision, Recall, F1-score, and Area under the Precision-Recall Curve (PR-AUC) [5, 6]. Continuous response metrics for binary classification such as Mean Absolute Error (MAE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE) have also been used. The second consists of multi-classification metrics that measure how well an algorithm predicted the amount of crime in each area. Common multi-classification metrics use multinomial logistic regression accuracy and loss functions [7, 8].

In criminology, various indices have been created to measure the overall proportion of crime captured in areas classified as hot-spots by an algorithm. Before 2008, hot-spot mapping models were mainly evaluated via visual inspection and calculating a hit-rate. These methods for evaluating hot-spot maps are easy to understand; however, they lacked consistency, were prone to gaming, and lacked meaning in their measure [9–11]. The prediction accuracy index (PAI) was developed to derive a less subjective approach and was the first crime index to specifically measure the "performance" of a crime hot-spot mapping model [10]. Recent crime indices include RRI, PEI, and PEI* [12 – 15]. Additionally, hot-spot maps have been measured based on their variability via the dynamic variability index (DVI) and compactness via the Area-Parameter ratio (AP) and clumpiness index [16]. Additional discussion of relevant statistical metrics and crime indices can be found in [3, 17]. Today, PAI remains the dominant measure used to analyze crime hot-spot mapping models in practice despite inconsistency across applications, methodological concerns, and best practices suggesting using multiple crime indices for evaluation purposes [12, 14, 16]; while the RRI, PEI, and PEI* are not regularly used they offer alternative policing-centric measures. Therefore, there is a need to further refine, discuss, and define new indices to clarify which metrics should be used in various contexts.

3. Introduction of a Standard Notation for Criminology Indices

No standard notation exists to define crime indices [14]. A standard notation helps unify commonalities between different crime indices. Therefore, we propose the following standard notation to describe and define hot-spot mapping indices in **Table 1**. We then apply this standard notation to the most common crime indices used in practice: PAI, RRI, PEI, and PEI*. This allows us to mathematically define the differences between them and reveal the optimization model behind PEI*. Creating this standard notation also aids future research in creating new metrics and indices.

Table 1: Proposed Notation of Hot-spot Mapping Indices

Sets	**Definition**
T	The set of equal length time periods considered in the study
P	The set of places (e.g., grid cells, street segments, patrol areas) where events (e.g., crimes, calls for service) are aggregated to
$\hat{P}_t$	The set of places is defined as hot-spots in time period $t \in T$. Note: $\hat{P}_t \subseteq P$
Parameters	**Definition**
a_p	The size (e.g., area, length) of place, $p \in P$
A	The total study area. Note: $A = \sum_{p\in P} a_p$
$\hat{A}_t$	The total area of all hot-spots identified in time period $t \in T$. Note: $\hat{A}_t = \sum_{p\in \hat{P}_t} a_p$
$n_{p,t}$	The number of events (e.g., crimes, calls for service) that occur in place, $p \in P$, during time period $t \in T$
N_t	The total number of events in the study area during time period $t \in T$. Note: $N_t = \sum_{p\in P} n_{p,t}$
$\hat{N}_{t_2,t_1}$	The total number of events during time period $t_2 \in T$ that are in hot-spots identified at time period $t_1 \in T$, where $t_2 \geq t_1$. Note: $\hat{N}_{t_2,t_1} = \sum_{p\in \hat{P}_{t_1}} n_{p,t_2}$
Decision Variable	**Definition**
x_p	Percentage of a place, $p \in P$, designated as a hot-spot
Objective	**Definition**
$\hat{N}^*_{t_2,t_2}$	The maximum number of events that occur in time period $t_2 \in T$ and in designated hot-spots where the total area of the hot-spots, $\hat{A}_{t_2}$, must be equal to or less than a total hot-spot area of $\hat{A}_{t_1}$, where $t_1 \in T$ and $t_2 \geq t_1$. Note: $\hat{N}^*_{t_2,t_2} = \max_x \sum_{p\in P} n_{p,t_2} x_{p,t_2}$

We now elaborate and demonstrate the notation of sets and parameters in **Table 1.** Let T be the set of equal length time periods considered in the study. Let A represent the total study area. There is no standard unit to define area, which depending on the application and data available, can vary from square miles to street lengths. To formally calculate A, we define P, as the set of places (e.g., grid cell, police beat, census block) where events (e.g., crimes, calls for service) are aggregated. We also define a_p as the size (e.g., length, area) of a place $p \in P$. Police departments typically use the size of a place as a proxy for required resources (e.g., during a week, a single police car could patrol 0.25 square miles). Therefore, each place may vary in size, a_p, but generally requires the same amount of resources to serve. We can now formally define the total study area as, $A = \sum_{p \in P} a_p$. If all places are of equal size, the subscript p can be omitted, and a then can represent the size of every individual place in the study area. We now formalize the notation for calculating the number of events (e.g., crimes, calls for service) and where they are located. Let $n_{p,t}$ be the number of events that occur in place, $p \in P$ during time period $t \in T$. Also, let N_t be the total number of events that occur in the study area during time period $t \in T$. Therefore, we can formally define $N_t = \sum_{p \in P} n_{p,t}$. Looking at hot-spots specifically, let $\hat{P}_t$ be the set of places that are identified as hot-spots at time period $t \in T$. We can now formally define $\hat{A}_t$ as the total area of all hot-spots identified in time period, $t \in T$, or $\hat{A}_t = \sum_{p \in \hat{P}_t} a_p$. Next, let $\widehat{N}_{t_2,t_1}$ represent the total number of events during time period $t_2 \in T$ that are in hot-spots identified at time period $t_1 \in T$, where $t_2 \geq t_1$. Therefore, $\widehat{N}_{t_2,t_1} = \sum_{p \in \hat{P}_{t_1}} n_{p,t_2}$. Lastly, let $\widehat{N}^*_{t_2,t_2}$ be the maximum number of events that occur in time period $t_2 \in T$ and in designated hot-spots where the total area of the hot-spots, $\hat{A}_{t_2}$, must be equal to or less than a total hot-spot area of $\hat{A}_{t_1}$, where $t_1 \in T$ and $t_2 \geq t_1$.Therefore, $\widehat{N}^*_{t_2,t_2} = \max_x \ \sum_{p \in P} n_{p,t_2} x_{p,t_2}$.

3.1 Validation of the Standard Notation

Table 2 is re-created from the computational example in [14]. We use the discussion from [14] to guide the conversion of the common crime indices PAI, RRI, PEI, and PEI* to our new notation in **Table 1**, resulting in the equations mapped in **Table 2**.

Table 2: Computational Example, where $\hat{A}_{t_1} \neq \hat{A}_{t_2}$, revised figure from [14] using new notation

	PAI	RRI	PEI	PEI*
Notes: $\hat{A}_{t_1} \neq \hat{A}_{t_2}$. Hot-spot (HS) locations are shaded gray if the grid has ≥ 2 events at the time the map was created.	$\dfrac{\left(\frac{\hat{N}_{t_2,t_1}}{N_{t_2}}\right)}{\left(\frac{\hat{A}_{t_1}}{A}\right)}$	$\dfrac{\left(\frac{\hat{N}_{t_2,t_1}}{N_{t_2}}\right)}{\left(\frac{\hat{N}_{t_1,t_1}}{N_{t_1}}\right)}$	$\dfrac{\left(\frac{\hat{N}_{t_2,t_1}}{\hat{A}_{t_1}}\right)}{\left(\frac{\hat{N}_{t_2,t_2}}{\hat{A}_{t_2}}\right)}$	$\dfrac{\hat{N}_{t_2,t_1}}{\hat{N}^*_{t_2,t_2}}$
(c) Instance 3	$A=9,\ \hat{A}_{t_1}=1,\ \hat{A}_{t_2}=2,\ N_{t_1}=2,\ N_{t_2}=5,$ $\hat{N}_{t_1,t_1}=2,\ \hat{N}_{t_2,t_2}=5,\ \hat{N}_{t_2,t_1}=3,\ \hat{N}^*_{t_2,t_2}=3$			
Time 1 HS Map 1: $\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix}$ Time 2 HS Map 1: $\begin{bmatrix}0&0&0\\0&3&2\\0&0&0\end{bmatrix}$ HS Map 2: $\begin{bmatrix}0&0&0\\0&3^*&2\\0&0&0\end{bmatrix}$	$\frac{3/5}{1/9}$ $= 5.4$	$\frac{3/5}{2/2}$ $= 0.6$	$\frac{3/1}{5/2}$ $= 1.2$	$\frac{3}{3}$ $= 1$
(d) Instance 4	$A=9,\ \hat{A}_{t_1}=3,\ \hat{A}_{t_2}=2,\ N_{t_1}=6,\ N_{t_2}=5,$ $\hat{N}_{t_1,t_1}=6,\ \hat{N}_{t_2,t_2}=4,\ \hat{N}_{t_2,t_1}=3,\ \hat{N}^*_{t_2,t_2}=5$			
Time 1 HS Map 1: $\begin{bmatrix}0&0&2\\0&2&0\\0&0&2\end{bmatrix}$ Time 2 HS Map 1: $\begin{bmatrix}0&0&1\\2&2&0\\0&0&0\end{bmatrix}$ HS Map 2: $\begin{bmatrix}0&0&1^*\\2^*&2^*&0\\0&0&0\end{bmatrix}$	$\frac{3/5}{3/9}$ $= 1.8$	$\frac{3/5}{6/6}$ $= 0.6$	$\frac{3/3}{4/2}$ $= 0.5$	$\frac{3}{5}$ $= 0.6$

Table 2 shows two separate instances described in [14]:

> "For the following examples, we use a naive hot-spot mapping algorithm that designates any grid cell with two or more events in a given time period as a hot-spot. Hot-spots are shaded gray. We review four separate instances, i.e., two in Table 2 and two in Figure 1. Each instance has a set of three grids containing nine cells each. The left grid in each instance lists the number of events that occur in each cell at time 1, where the shaded cells are the hot-spots designated based on the events in time 1. The top right grid in each instance lists the number of events that occur in each cell at time 2, where the shaded cells are the same hot-spots that were designated in time 1. Lastly, the bottom right grid in each instance shows the number of events that happened in each cell at time 2, where the shaded cells are the hot-spots that are designated based on the events in time 2. The cells with an asterisk (*) indicate the cells used to calculate $[\widehat{N}^*_{t_2,t_2}]$. The PAI, RRI, PEI, and PEI* are calculated for each instance. In order to compare between instances and across all crime indices, we assume each instance is the same jurisdiction."

Interpretation of the indices can be found in [14]. The resulting calculations of each of the crime indices are equivalent to the original calculations in [14]. By putting these crime indices into a standard flexible notation, we can better identify the differences between indices. Additionally, we can now see the underlying optimization model behind PEI*, specifically the calculation of $\widehat{N}^*_{t_2,t_2}$.

4. Connecting a Crime Index to Optimization: A Continuous Knapsack Problem

We now show the continuous knapsack problem used to find $\widehat{N}^*_{t_2,t_2}$. In practice, the total hot-spot area represents the limited available resources that can be deployed for a strategy/tactic over a specific time period. For example, a police department's budget may allow police to patrol or provide community outreach to 10 areas. The chosen areas may change from week to week or month to month, but the total available resources do not typically change between micro time levels. Therefore, finding $\widehat{N}^*_{t_2,t_2}$ can be modeled as a continuous knapsack problem where the decision variables represent which areas can be classified as hot-spots for a given time period, the objective is to maximize the number of events in classified hot-spot areas, and the knapsack constraint that hot-spot areas can only sum up to a given total hot-spot area, $\hat{A}_{t_1}$.

Formally, we first define Equation (3) as the set of decision variables, x_p as the percentage of a place, $p \in P$, designated as a hot-spot. Typically, x_p will be 1 or 0, which means that location p is or is not designated as a hot-spot area, respectfully. However, we allow x_p to take fractional values in the case where all spatial units are not of equal size (e.g., trimmed cells at boundary lines) and resulting in the need for a partial spatial unit to be included as a hot spot to meet the constraint of $\hat{A}_{t_1} \geq \hat{A}_{t_2}$. This simplification also makes sense in practice since police officers could provide a fractional number of services (i.e., hours) to a specific area. We define the objective function as Equation (1), where we find the maximum number of events at time period $t_2 \in T$, in optimal hot-spots, by varying the values of the x decision variables. Equation (2) is the knapsack constraint which shows the total chosen hot-spot area must be less than or equal to a given amount of area that can be provided resources (i.e., $\hat{A}_{t_1}$).

$$\widehat{N}^*_{t_2,t_2} = \max_x \sum_{p \in P} n_{p,t_2} x_p \tag{1}$$

$$subject\ to \quad \hat{A}_{t_1} \geq \sum_{p \in P} a_p x_p \tag{2}$$

$$x_{p,t_2} \in [0,1] \; \forall p \in P \tag{3}$$

The optimization model represented by equations (1)-(3) is a continuous Knapsack problem and, therefore, can be solved in polynomial time via the greedy algorithm [18]. While the greedy algorithm is not novel, its application to solve for the optimal number of events a hot-spot map with limited resources could capture is. To help contextualize this application of the greedy algorithm, we step through it in the algorithm outlined in **Figure 1**, using the notation in **Table 1**. The algorithm will return the maximum number of events in places that can be designated as hot-spot areas in time period $t_2 \in T$ (i.e., $\widehat{N}^*_{t_2,t_2}$). The algorithm also requires the following inputs: the number of events that occur in each place $p \in P$ at time period $t_2 \in T$ (i.e., n_{p,t_2}), the size of each place $p \in P$ (i.e., a_p), and the total amount of area we can assign as hot-spots (i.e., $\hat{A}_{t_1}$ where $t_1 \in T$ and $t_1 \leq t_2$). In Step 1, we initialize *Size* and $\widehat{N}^*_{t_2,t_2}$ to equal zero. The variable *Size* will keep track of the current hot-spot area size. In Step 2, we calculate the density ratio d_{p,t_2} for each place, $p \in P$, where $d_{p,t_2} = n_{p,t_2}/a_p$. In Step 3, we sort the density ratios from largest to smallest. In Step 4, we execute the greedy algorithm which involves adding events from the places with the highest density ratios until the size of the added places equals our input of the maximum hot-spot area, $\hat{A}_{t_1}$, allowing us to return the corresponding final $\hat{N}^*_{t_2,t_2}$. In the case where $Size < \hat{A}_{t_1}$, but adding the next area would cause $Size + a_p > \hat{A}_{t_1}$, we calculate the percentage of the next place such that the total area equals $\hat{A}_{t_1}$ i.e. $(\hat{A}_{t_1} - Size)/a_p$ .We then multiply this same percentage by events in that place, n_{p,t_2} and add it to obtain our final $\hat{N}^*_{t_2,t_2}$. Using this procedure for the previous example in Table 2, we can define $a_p = 1\ \forall p \in \{1,..9\}$. It can then be shown that $\hat{N}^*_{t_2,t_2}$ equals 3 and 5 for instances 3 and 4, respectively.

Algorithm 1 Greedy Algorithm for $\hat{N}^*_{t_2,t_2}$

Input: t_1, t_2, $a_p\ \forall p \in P$, $\hat{A}_{t_1}$, and $n_{p,t_2}\ \forall p \in P$
Output: $\hat{N}^*_{t_2,t_2}$

Size $= 0$ ▷ **STEP 1.** Initialize
$\hat{N}^*_{t_2,t_2} = 0$
for $p \in P$ **do** ▷ **STEP 2.** Calculate density ratios
    $d_{p,t_2} = (n_{p,t_2})/a_p$
end for
$L = \text{Sort}(P \text{ by } d)$ from largest to smallest ▷ **STEP 3.** Sort density ratios
for every $l \in L$ **do** ▷ **STEP 4.** Execute Greedy Algorithm
    if $\hat{A}_{t_1} > \text{Size} + a_l$ **then**
        $\text{Size} = \text{Size} + a_l$
        $\hat{N}^*_{t_2,t_2} = \hat{N}^*_{t_2,t_2} + n_{l,t_2}$
    else
        $\hat{N}^*_{t_2,t_2} = \hat{N}^*_{t_2,t_2} + (\frac{\hat{A}_{t_1} - \text{Size}}{a_l}) \cdot n_{l,t_2}$
        Return $\hat{N}^*_{t_2,t_2}$
        Exit
    end if
end for

Figure 1: Algorithm 1: Greedy Algorithm for $\hat{N}^*_{t_2,t_2}$

The algorithm can be solved in linearithmic time O(n log n) for the sort and an additional O(n) for the computation of n. We emphasize that the optimization model is not meant to assign the hot-spot weight of the cell [0,1] in practice, but is used to solve for $\hat{N}^*_{t_2,t_2}$ which is needed to calculate the criminology metric PEI*. The value of $\hat{N}^*_{t_2,t_2}$ has been used as a proxy for how the "best" hot-spot map that accounts for the restricted amount of available resources would have performed [14, 18]. We recognize the objective function (2) used to calculate $\hat{N}^*_{t_2,t_2}$ may not be the only way to cacluate how a "best" hot-spot map would reasonably perform. For example, an optimal $\hat{N}^*_{t_2,t_2}$ may involve spread out hot-spot assignments or be overly optimized for specific places, where a map that has more clustered hot-spots may be more desirable for police departments to use. A more rigorous analysis of PEI* that demonstrates these trade-offs is needed, as well as a more thorough analysis of how alternative objective functions may affect how various hot-spot maps perform.

5. Conclusion

Hot-spot maps are used by the majority of police departments to make strategic and operational decisions. Currently, there are many methods to generate hot-spot maps, yet there lacks a consensus on which methods are "best." We argue this is due to a lack of discussion of and diverging definitions on what makes a hot-spot map "good," i.e., what metrics should be used to evaluate them. We review how statistical measures define performance through precision/error and demonstrate how crime indices, used in practice, focus on the relative effectiveness or efficiency of a hot-spot map. Therefore, standardizing the evaluation of hot-spot maps is critical in determining the "best" hot-spot map-generating methods. Before we standardize, research needs to be done to address what makes a hot-spot map "good" (e.g., accurate, fair, ethical [19], operationally efficient), which may depend on the use of the hot-spot map. Analysis of potential trade-offs of opposing objectives and the development of metrics that balance these complexities is needed. Lastly, we provide a motivational example of how the criminology index, PEI*, innately contains a continuous knapsack optimization problem. We hope the fields of operations research and optimization, which are well-suited to analyze complex decisions, play a greater role in the future of hot-spot map evaluation.

Disclaimer

The opinions, findings, conclusions, and recommendations expressed in this publication are those of the authors and do not necessarily reflect the official position or policies of the U.S. Department of Justice.

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